PREFACE

To the Student

This textbook is written for you. That means it is written so that you can read it and understand it. Of course, reading a mathematics textbook is not the same as reading a novel; when reading mathematics you will often have to stop, think about the material, do some examples, and reread the material again. In this book we will spend a lot of time on algebra and precalculus topics; this will give you the knowledge and confidence you need to succeed in calculus, regardless of your previous background.

This textbook encourages you to understand calculus, rather than just memorize facts or equations. You will be responsible for understanding *why* the mathematics works, not just *how* to apply it. In the homework exercises, "Concepts" problems will help you test your understanding of the material, "Skills" and "Applications" exercises will help you apply that material to computational and real-world problems, and the exercises in the "Proofs" section will help you review the important proofs from the reading and ask you to extend the theory beyond what is given in the section.

Suggestions for Students

- Read each section carefully, and test your understanding as you go by answering the "Questions" in the reading. Read the material before attempting the exercises.
- Pay careful attention to the "Cautions" given in the reading, as they will help you avoid common mistakes.
- Each section of the homework exercises begins with a "Problem 0" that asks you to write your own summary of the material; this summary will help you organize your thoughts and prepare for quizzes and tests.
- Answers for about half of the homework exercises are given in the back of the book. It is good practice to check your own answers; the "Checking" notes in the reading will help you do this.

To the Instructor

This textbook is different from other calculus books on the market for three reasons: First, it integrates calculus with precalculus and algebra material in a manner suitable even for math and science majors. Second, it is written in a way that students can understand (without watering down the material). Third, the homework exercises hold students accountable for the theory and the reading and test conceptual understanding as well as computational skill.

The exercises in this book address two problems common to many calculus courses: first, that students believe that everything they need to know is covered in the homework exercises, and second, that students often complain that quizzes and tests do not reflect the homework problems they did. *Integrated Calculus* includes many homework problems that ask students to explain definitions, concepts, theorems, or proofs from the reading, as well as problems that test their computational skills and problems that apply what they have learned to real-world situations. There is also a "Problem 0" in each homework section that asks students to summarize the material. If you like, you can use this homework problem to test whether students read the section before class or, alternatively, to test whether the students have understood the material by the following day.

What Is Integrated Calculus?

Integrated Calculus is designed for a two-semester course that integrates differential and basic integral calculus with precalculus and algebra material. The material includes all of what is commonly referred to as "Calculus I," some of the material known as "Calculus II," and a good deal of algebra and precalculus material. All material, even the algebra and precalculus, is handled in a fashion that is appropriate for math and science majors.

This textbook has four major parts. Part I covers elementary algebra, logic, and proofs and the concepts of functions, limits, and derivatives. The focus is on concepts rather than calculations; most examples are computationally simple or based on graphs. Parts II and III of the text use the concepts and techniques learned in Part I to investigate the properties of algebraic and transcendental functions. Calculations get progressively more difficult as students progress through the types of functions in Parts II and III. Part IV covers basic integration skills and applications.

Goals and Features of the Text

- **A readable text for students** This book is meant for students to read. The text is readable while still being rigorous and is full of examples, questions, and cautions. Proofs in the text are written with the student (rather than just the instructor) in mind. The book is also organized so as to be a good reference text when a student needs to review a topic.
- **Integrates calculus, precalculus, and algebra in a meaningful way** Calculus, precalculus, and algebra are integrated throughout this textbook. Precalculus and algebra are used to investigate calculus, and calculus is used to investigate precalculus (specifically graphical features of functions).
- **Inverted structure** From *Integrated Calculus,* students will learn the basic theories and techniques of calculus and then use this knowledge to explore different types of functions (power, polynomial, exponential, and so on). This approach differs from the usual structure of a calculus book where students learn one technique or concept after another, each time applying their knowledge to *all* types of functions.
- **Systematic development of calculation skills** The first part of the book covers functions, limits, and derivatives through graphic and simple calculation examples, concentrating more on concepts than on symbolic manipulation. In the second and third parts of the book, students apply their knowledge to more and more complicated types of algebraic and transcendental functions, building calculation skills along the way.
- **Beneficial repetition** Because of the inverted structure of this book, students will revisit topics many times as they study different types of functions. For example, "definition of derivative" problems don't appear just in one section, they appear in each chapter that investigates a certain class of functions.
- **Focus on algebra** Many calculus students struggle with even simple algebraic manipulations. *Integrated Calculus* covers algebraic material from simple factoring and manipulation of exponents to complex techniques such as polynomial long division and manipulation of sums in sigma notation.
- **Rigorous treatment and encouragement of mathematical thought** Although *Integrated Calculus* covers basic topics in precalculus and algebra, it is not a "dumbed-down" or "slow" treatment of calculus. Every topic is treated rigorously; students learn about logic and proofs early in the text and use these skills

throughout. Proof by induction, delta-epsilon proofs, and general Riemann sums are included in the reading and exercises as well as many short, easy proofs.

- **Exercises that test more than computational skills** The exercises in *Integrated Calculus* hold students accountable for the reading by asking conceptual as well as computational problems. Each exercise set begins with "Concepts" questions that test the students' knowledge of definitions, theorems, and concepts from the reading. The remaining exercises are divided into "Skills," "Applications," and "Proofs" sections.

Comments and Suggestions for Instructors

- This textbook is written with an "inverted structure." *Integrated Calculus* applies each skill to all types of functions in contrast to most calculus textbooks that are organized by skills; that is, they cover one skill after another (for example, differentiation) and apply each skill to all types of functions. In Part I students learn the basic skills they need for the course. Then in each section of Parts II and III, they use these skills to investigate particular types of functions (for example, polynomial functions). This "inverted structure" provides for a lot of beneficial repetition; for example, students see the definition of derivative over and over again as they study different types of functions. This approach also allows for a systematic development of calculational skills. Students first see the definition of derivative in a simple calculational context. Then, as the textbook progresses through more complicated types of functions, they apply the definition of derivative in more challenging contexts. This is particularly helpful to students with a weak background in algebra.
- Limits and derivatives are introduced using only simple polynomial functions, piecewise functions, and graphical examples. The power rule is introduced in Chapter 3 using only positive integer powers; in Chapter 4 it is extended to negative integer powers and rational powers; then the general power rule is proved in Chapter 7. The quotient rule is presented in Chapter 6, and the product and chain rules presented in Chapter 7. This allows for a careful development of differentiation rules and is reflected in exercises that ask students to identify whether a given function can be differentiated given the rules developed so far.
- Other topics also get covered with increasing complexity as the book progresses. Optimization is presented first in Chapter 5 using polynomial functions, and then in Chapter 7 with general algebraic functions. Curve sketching gets progressively more sophisticated as more complicated types of functions are introduced. Similarly, algebraic techniques, domain computations, limits, and derivative calculations progress from simple to challenging throughout the book.
- Because this calculus course is geared toward students with weak algebra and precalculus backgrounds, logarithmic functions are defined as inverses of exponential functions rather than the more rigorous definition in terms of integration.
- Part IV of *Integrated Calculus* includes more integration topics than are normally included in a "Calculus I" course. This provides the background and experience with integration that is needed by students in science and math-related majors who might not need all of "Calculus II," but who do need more integration techniques than are typically covered in "Calculus I."
- It is possible to cover all sections of the book in a two-semester sequence of courses. However, it is also possible to customize the course for various needs. Certain sections can be omitted if you prefer. For example, the integration techniques in Sections 14.2, 14.3, and 14.4 can be skipped without much

trouble; even Chapter 15 is covered later (homework problems will have to be selected carefully). You might also consider Sections 4.5, 7.4, 11.1, and 11.2 to be optional (assuming you will skip Sections 14.4, 15.1, 15.2, 15.3, or 15.4). For a less rigorous course, you could omit much of the section on proofs (Section 0.5) and the section on delta-epsilon proofs (Section 2.2). You could also combine two or more sections into one lesson by omitting some proofs. As one example, the sections on limits and derivatives of exponential functions (Sections 8.3 and 8.4) could be combined into one lesson if you were to omit the proofs concerning the continuity of and differentiation rules for exponential functions.

- Throughout the textbook, students are encouraged to use graphing calculators to check their answers and investigate graphs of functions. However, it is also possible to use this book in a course that does not require graphing calculators. In addition, all references to calculators in this textbook are nonspecific, that is, they do not assume any particular type of graphing calculator. Any calculator that can graph functions and approximate extrema, points of intersection, slopes, and areas should be sufficient.

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Comments and Suggestions

I welcome comments and suggestions from instructors and students using this textbook. In particular, I am interested in hearing about any typographical, mathematical, or formatting errors you find in this book; any topics or sections you would like to see added to the book; any topics or sections you think could be improved; and any pedagogical or stylistic comments. I would also be happy to answer any questions you might have about this book or the type of course it is for. I can be reached by email at taal@math.jmu.edu or by post at

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